

MTH 203, Calculus III, Quiz One Spring 2013

Ayman Badawi

QUESTION 1. a) Find a vector that is parallel to < -2, 4, 2 > but has length 6.
b) Find the angle between the two vectors $U = i + \sqrt{3}j$ and $V = i + 3j - k$.

QUESTION 2. Find an equation of the plane that passes through the points $(1, 0, 2)$, $(2, 1, 2)$ and $(1, 1, 2)$. Does the line : $x = 1 + t, y = 3 - 2t, z = 5 + 4t$ lie in the plane ? explain.

QUESTION 3. Find a parametric equations of the line that passes through the point Q = (1, -1, 10) and perpendicular to the plane P : $3x - 4y = 22$.

QUESTION 4. Find an equation of the plane P where each point in P is equidistant from the two points (1, 2, -3) and (3, 2, 7).

QUESTION 5. Find an equation of the plane P that contains the line $< t - 1, 2t + 3, 5 >$ and the point $(4, 2, -1)$. Does P contain the vector $< 10, -2, -12 >$? explain

QUESTION 6. Given $Z = f(x, y) = x^2e^{xy} + \sqrt{4x - 2} + \frac{7}{\sqrt{4-y}} + y^3 - x^2 + 3xy$

a) Find the domain of $f(x, y)$. b) Find $f_x = dz/dx$

QUESTION 7. Given $ln[z(x + y)(x + 2y)^3] + 10x^2 + y^3 - 4xy = 11$. Find dz/dx when $x = 1, y = 0$.

QUESTION 8. Find the partial derivative dz/dy for $e^z = xyz + yz^2 + xln(y) + 3z + 2x$

QUESTION 9. (i) Find the volume of the solid object that has a basis, say D, in the xy-plane where (0, 0), (1, 1), (1, 2) , and (0, 1) are the vertices of D and the height z is a function in terms of x and y where $z = 2x + 2$.

(ii) Find the volume of the solid subject that has a basis consists of all points in the upper half of the xy-plane that are enclosed between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$, and the height z is given as a function in terms of x and y where $z = 4x^2 + 4y^2$.

(iii) Find the surface area of the solid subject $z = x^2 + 3y$ over the region , say D, where D consists of all points in the first quadrant of the xy-plane that are enclosed between the two graphs $y = x^2 + 24x$ and $y = x^2$ where $0 \le x \le 2$.

QUESTION 10. Given the force field $F(x, y) = (1 + 2xy)i + (x^2 - 3y^2)j$
Is $F(x, y)$ conservative? Explain.

(i) A particle moves along line segments from (0, 0) to (4, 1) to (3, 4) to (2, 2) (counter clock-wise). Find the work done by the above force $F(x, y)$ in moving the particle along the given line segments from (0, 0) to (2, 2)

(ii) Let C be the part of the curve of the ellipse $x^2 + y^2/4 = 1$ in the second quadrant of the xy-plane, and assume that C is positively oriented. Find the area of the side that is bounded between the function $z = -9xy$ and C.

QUESTION 11. Find the volume of the solid subject that has a basis consists of all points in the upper half of the xy-plane that are enclosed between the circle $x^2 + y^2 = 4$ and the line $y = \sqrt{2}$, where the height $z = y$.

QUESTION 12. a) Given the force field $F(x, y) = yi + 5xj$. A particle moves along line segments from (0, 3) to (1, 3) to (4, 6) to (0, 6), then back to (0, 3) (counter clock-wise). Find the work done by the force $F(x, y)$ in moving the particle along the given line segments from (0, 3) back to (0, 3). [Hint: Recall that if $F = A(x, y)i + B(x, y)j$, then $\int_C F \cdot dr = \int_C A(x, y)\, dx + B(x, y)\, dy$]
b) The same as in (a) but assume that the particle moves along line segments from $(1, 3) to (0, 3) to (0, 6) to (4, 6)$, then back to $(1, 3)$

QUESTION 13. Find the area of the side that is bounded by the curve: $x = e^t + e^{-t}, y = 2t, 0 \le t \le 1$ and $z = x^2y$

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MTH 203, Calculus III, Exam I, Spring 2013

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QUESTION 1. Let D be the region in the first quadrant of the xy-plane that is bounded by $y = x^2 - 2$, x-axis, y-axis and $y = 2$. Find the surface area of the part of $z = x + \frac{2}{3}y^{3/2} + 10$ that is over the region D.

QUESTION 2. Let D be the region in the first quadrant of the xy-plane that is above the line $y = 1/2$ and below the circle $x^2 + y^2 = 1$. Find the volume of the object that has D as a basis and its height is determined by $z = \frac{2x}{x^2+y^2}$

QUESTION 3. Given the force field $F(x, y) = (1 + 2x + ye^x)i + (1 + 2y + e^x)j$
Is $F(x, y)$ conservative? Explain.

(i) A particle moves along line segments from (0, 1) to (2, 2) to (3, 4) to (0, 2) (counter clock-wise). Find the work done by the above force $F(x, y)$ in moving the particle along the given line segments from (0, 1) to (0, 2)

(ii) Given the force $F(x, y) = 6yi + 9xj$. A particle moves along the circle $x^2 + y^2 = 1$ from the point (1, 0) to (0, 1). Then it moves along line segments from (0,1) to (-1, 0) and back to (1, 0). Find the work done by the force $F(x, y)$

QUESTION 4. Given the force $F(x, y) = \frac{1}{x+1} i + 2xyj$. A particle moves along the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ from the point (2, 0) to (0, 3). Find the work done by the force $F(x, y)$.

QUESTION 5. Find the area of the side that is bounded by $z = -36xy$ and the curve $x^2 + y^2/4 = 1$, where $-1 \le x \le 0$ and $0 \le y \le 2$ (i.e., the curve is part of the ellipse $x^2 + y^2/4 = 1$ in the second quadrant of the xy-plane).

QUESTION 6. a) Find the equation of the plane that contains the points $(1,-2,4),(1,3,5),(2,-2,8)$.

b)Find a vector W that is parallel to $V = i + 2j + 2k$ but of length 7.25.

c) Find dz/dx when $x = 2$ and $y = 0$ where

$$e^{zxy} + 4zx^2 - z^2 + xz^2 - 6z + y^2 - zx^3 = 0$$

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MTH 203, Calculus III, Questions Spring 2013

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QUESTION 1. Let $f(x,y) = xln(y-2) + ye^{(2x-4)} + xy$.

a) Find the domain of $f(x,y)$.

b) Find the equation of the tangent plane to the solid object determined by f(x, y) at the point (2, 3).

c) Use (b) to approximate f(2.2, 2.8).

d) at the point (2, 3), in which direction does the rate of change of z obtain its maximum? What is the maximum?

e) Find the equation of the plane that contains the vector $\nabla f|_{(2,3)}$ and the two points (2, 3), (2, 3, f(2, 3)) and show it is perpendicular to the xy-plane.

f) The plane in (e) intersects $f(x,y)$ in a curve C. Find a parametric equations of C.

f/2) Find a parametric equations of the tangent line to the curve C (above) when $x = 2$ (note (2, 3) is in the domain of C. .

f/3) Find the slope of the line in (f/2) in the direction of the vector $\nabla f|_{(2,3)}$.

f/4) Are you surprised? What is connection between the number you obtained in (f/3) and (d)?

f/5) NICE!!! The plane in (b) intersects the plane in (e) in a line L ? find a parametric equations of the line L? Any comments!!! Can you relate L to the line in (f/2)?

QUESTION 2. Given the curve: $x \in R, y = -2x+3, z = sin(x)$.

a) Sketch the curve.

a/2) Find a parametric equations of the tangent line to the curve when $x = \pi/6$.

a/3) Find the angle between the directional vector of the line in (a/2) and the vector $v = 2i - 4j$. Then find the tangent of such angle.

a/4) Find the directional derivative of z as above in the direction of the vector $v = 2i - 4j$.

a/5) Do you observe any connection between (a/4) and (a/3).

QUESTION 3. The solid object $x^2 + y^2 + z = 18$ intersects the plane $2x + y = 0$ in a curve C. Find a parametric equations of the curve C.

QUESTION 4. Given : $3ze^{zx} + ln(y) - yz + zx = 20$, $x = 3t + 2s - 7$, $y = 4t - s - 1$. Find dz/dt and dz/ds when $t = 1$, $s = 2$.

QUESTION 5. Sketch the curve C: $x \in R, y = 2x+1, z = 1 + 0.5e^{2x}$. Find the arch length of C for $0 \le x \le ln(2)$.

QUESTION 6. Find all points in the xy-plane where the rate of change of z is maximum in the direction of the vector -3i + 4j, where $f(x,y) = 2x^2 + y^2 - xy + 4$

QUESTION 7. a) Find a parametric equations of the line that is perpendicular to the plane $3x - 2y + 4z = 6$ and intersects the plane at the point $(0, 1, 2)$

b) Find an equation of the plane P where each point in P is equidistant from the two points $(1, -4, 5)$ and $(3, -2, 1)$.

b/2) Choose a point in the Plane as in (b). Find a parametric equations of the line L that contains the selected point and each point in L is equidistant from the two points $(1, -4, 5)$ and $(3, -2, 1)$.

c) Find an equation of the plane P that contains the line $< t-1, 2t+3, 5 >$ and the point $(4, 2, -1)$. Does P contain the vector $< 10, -2, -12 >$? explain

QUESTION 8. a) Given a, b, c, d are some constants where $< at, bt, b >$ is the line of intersection of the two planes $cx + dy + dz = -4$ and $2x - 3y + 2z = 8$. Find the values of a, b, c, d.

b) Given that the two planes, $P_1 : 2x + y + z = 1$ and $P_2 : -2x + y + 3z = -4$ intersects in a line L.

i) Find a parametric equations of L.

ii) Find the distance between the point $(2, 2, 0)$ and P_1.

iii) Find the distance between $(2, 2, 0)$ and L

QUESTION 9. a) The two objects $4x^2 + 9y^2 = 1$ and $z = xy + 2$ intersects in a curve (vector function) $r(t)$. Find a parametric equations of $r(t)$.

b) Let $r(t) =< 2t+1, \dfrac{e^t - ln(t+1) - 1}{cos(t) - 1}, t+1 >$

i) Find the domain of $r(t)$ in terms of t.

c) Find the arc-length of $r(t) =< 2e^t, 3e^{-t}, \sqrt{12}t >$ when t is between 0 and $ln(0.5)$.

QUESTION 10. a)Find the area of the triangle that has the vertices $(1,2),(1,4),(0,2)$.

b) Given the three vectors (having the same initial point), $V =< 2,2,0 >, W =< 1,-2,0 >$, and $D =< 1,1,-2 >$. Do they lie in the same plane?

c) Find the equations of the tangent line to the curve $r(t) =< sin(2t), 2cos(2t), \sqrt{3}sin(2t) - 2\sqrt{3} >$ at the point that is determined by letting $t = \pi/4$

QUESTION 11. a)Let $f(x,y) = (y^2+4)e^{x^2} - 10y + 1$ Find the critical points of $f(x,y)$. Does $f(x,y)$ have local min. (max) values? if yes then find them.

a/2) Let $f(x,y)$ as above defined on the rectangle that has the vertices $(0,0),(4,0),(4,10),(0,10)$. Find the absolute max and the absolute min of $f(x,y)$ over the rectangle.

a/3) Given the force field $F(x,y,z) = (2x+z)i + 2yj + (2z+x)k$. Is $F(x,y,z)$ conservative? If yes, find a function $L(x,y,z)$ such that $\nabla L(x,y,z) = F(x,y,z)$.

a/4) A particle moves along line segments from (0, 0, 3) to (1, 0, 3) to (4,0, 6) to (0, 0, 6). Find the work done by the above force $F(x,y,z)$ in moving the particle along the given line segments from (0, 0, 3) to (0, 0, 6).

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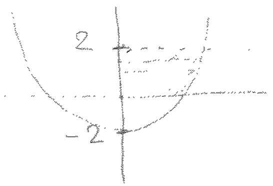


MTH 203, Calculus III, Exam I, Spring 2013

Ayman Badawi

$x^2 = y + 2$
$x = \sqrt{y+2}$

QUESTION 1. Let D be the region in the first quadrant of the xy-plane that is bounded by $y = x^2 - 2$, x-axis, y-axis and $y = 2$. Find the surface area of the part of $z = x + \frac{2}{3}y^{3/2} + 10$ that is over the region D.

$\int\int_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$, $f_x = 1$, $f_y = \frac{3}{2}\frac{2}{3}y^{\frac{1}{2}}$

$\int\int \sqrt{2+y}\, dA$

$\int_0^2 \int_0^{\sqrt{y+2}} \sqrt{2+y}\, dx\, dy$

$\int_0^{\sqrt{y+2}} \sqrt{2+y}\, dx = \sqrt{2+y}\, x \Big]_0^{\sqrt{y+2}} = \sqrt{2+y}\sqrt{2+y} = (2+y)$

$\int_0^2 2 + y\, dy$

$= 2y + \frac{y^2}{2}\Big]_0^2$

$= 4 + 2 = 6$ unit^2

QUESTION 2. Let D be the region in the first quadrant of the xy-plane that is above the line $y = 1/2$ and below the circle $x^2 + y^2 = 1$. Find the volume of the object that has D as a basis and its height is determined by $z = \frac{2x}{x^2+y^2}$

$\frac{\pi}{6}$, $\frac{1}{2}$

change to polar: $\theta_1 \rightarrow \cos\theta = \frac{1/2}{1} = 60°$ $\theta = \frac{\pi}{6}$

$\theta_2 \rightarrow \theta = 90 = \frac{\pi}{2}$

$R_2 = 1$

$y = \frac{1}{2}$

$R\sin\theta = \frac{1}{2}$

$R_1 = \frac{1}{2\sin\theta}$

$z = \frac{2R\cos\theta}{R^2\cos^2\theta + R^2\sin^2\theta}$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\frac{1}{2\sin\theta}}^{1} \frac{2R^2\cos\theta}{R^2\cos^2\theta + R^2\sin^2\theta}\, dR\, d\theta$

$\int_{\frac{1}{2\sin\theta}}^{1} 2\cos\theta\, dR = 2\cos\theta\, R\Big]_{\frac{1}{2\sin\theta}}^{1} = 2\cos\theta - \frac{\cos\theta}{\sin\theta}$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\cos\theta - \frac{\cos\theta}{\sin\theta}\, d\theta = 2\sin\theta - \ln(\sin\theta)\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= 2 - 1.693$

$= 0.307$ unit^3

A(x,y) B(x,y)

QUESTION 3. Given the force field $F(x,y) = (1 + 2x + ye^x)i + (1 + 2y + e^x)j$
Is $F(x,y)$ conservative? Explain.

$A_y = e^x$, $B_x = e^x$

$A_y = B_x$ ∴ conservative
∴ work is independent of path of particle
∴ there is $L(x,y)$ such that $L_x = A$, $L_y = B$
and so fundamental theorem of line integral is applicable

(i) A particle moves along line segments from (0, 1) to (2, 2) to (3, 4) to (0, 2) (counter clock-wise). Find the work done by the above force $F(x,y)$ in moving the particle along the given line segments from (0, 1) to (0, 2)

find $L(x,y)$: $\int A(x,y)\,dx = \int 1 + 2x + ye^x\,dx = x + x^2 + ye^x$

$\int B(x,y)\,dy = \int 1 + 2y + e^x\,dy = y + y^2 + ye^x$

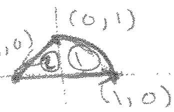
$L(x,y) = x + x^2 + ye^x + y + y^2$

$W = \int_C F \cdot dr = L(Q_2) - L(Q_1)$

$L(Q_2) = L(0,2) = 0 + 0 + 2 + 2 + 4 = 8$
$L(Q_1) = L(0,1) = 0 + 0 + 1 + 1 + 1 = 3$

$= 8 - 3$
$= 5$ units of work ✓

(ii) Given the force $F(x,y) = 6yi + 9xj$. A particle moves along the circle $x^2 + y^2 = 1$ from the point (1, 0) to (0, 1). Then it moves along line segments from (0,1) to (-1, 0) and back to (1, 0). Find the work done by the force $F(x,y)$



conservative? $A_y = 6$, $B_x = 9$ ≠ not conservative

apply green theorem:

$\iint_D B_x - A_y\,dA = \iint_D 9 - 6\,dA = \iint_D 3\,dA = 3 \times \text{Area of region}$

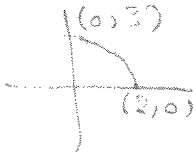
area of region: area① + area②

$= \frac{1}{4}\pi(1)^2 + \frac{1}{2}(1)(1)$

$= \frac{\pi}{4} + \frac{1}{2}$

$= 1.2854$ unit² ✓

Work $= 3 \times 1.2854$
$= 3.8562$ unit of work ✓



QUESTION 4. Given the force $F(x,y) = \frac{1}{x+1}i + 2xyj$. A particle moves along the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ from the point (2, 0) to (0, 3). Find the work done by the force $F(x, y)$.

conservative? $A_y = 0 \neq B_x = 2y$ not conservative.

r(t):
$x = 2\cos(t)$, $y = 3\sin(t)$, $0 \le t \le \frac{\pi}{2}$

$r(t) = \langle 2\cos(t), 3\sin(t) \rangle$

$dr = \langle -2\sin(t), 3\cos(t) \rangle$

$F = \frac{1}{2\cos t + 1} i + 2(2\cos t)(3\sin t) j$

$= \frac{1}{2\cos t + 1} i + 12\cos t \sin t$

$= \langle \frac{1}{2\cos t + 1}, 12\cos t \sin t \rangle$

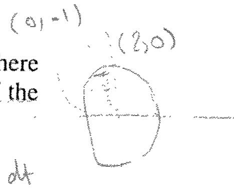
$\int_0^{\pi/2} F \cdot dR = \int_0^{\pi/2} \frac{-2\sin t}{2\cos t + 1} + 36\cos^2 t \sin t \, dt$

$= \ln(2\cos t + 1) - \frac{36}{3}\cos^3 t \Big]_0^{\pi/2}$

$= 0 - 0 - 1.0986 + 12$

$= 10.9014$ unit of work

QUESTION 5. Find the area of the side that is bounded by $z = -36xy$ and the curve $x^2 + y^2/4 = 1$, where $-1 \le x \le 0$ and $0 \le y \le 2$ (i.e., the curve is part of the ellipse $x^2 + y^2/4 = 1$ in the second quadrant of the xy-plane).



$\int_C Z\sqrt{x'^2 + y'^2}\, dt$

r(t):
$x = \cos t$, $y = 2\sin t$, $\frac{\pi}{2} \le t \le \pi$

$x' = -\sin t$
$y' = 2\cos t$

$z = -36(\cos t)(2\sin t) = -72\cos t \sin t$

$\int_{\pi/2}^{\pi} -72\cos t \sin t \sqrt{\sin^2 t + 4\cos^2 t}\, dt$

derive
$= 2\sin t\cos t - 8\cos t\sin t$
$= \cos t\sin t(2-8)$
$= \cos t\sin t(-6)$
$= -6\cos t\sin t$

$\frac{12 \times 2}{3}(\sin^2 t + 4\cos^2 t)^{3/2} \Big]_{\pi/2}^{\pi}$

$64 - 8 = 56$ unit^2

QUESTION 6. a) Find the equation of the plane that contains the points $(1, -2, 4), (1, 3, 5), (2, -2, 8)$.

choose as initial $(1,-2,4)$; $V_1 \to (1,3,5)$; $V_2 \to (2,-2,8)$

$V_1 = \langle 0, 5, 1 \rangle$

$V_2 = \langle 1, 0, 4 \rangle$

get normal vector

$$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ 0 & 5 & 1 \\ 1 & 0 & 4 \end{vmatrix} = 20i - (-j) + (-5k) = 20i + j - 5k \;(N)$$

assume point (x, y, z) in the plane, creating vector $\langle x-1, y+2, z-4 \rangle$ (V)

$N \cdot V = 0$ (perpendicular)

$\langle 20, 1, -5 \rangle \langle x-1, y+2, z-4 \rangle = 0$

$20(x-1) + (y+2) - 5(z-4) = 0$ standard form

$20x + y - 5z - 20 + 2 + 20 = 0$

$20x + y - 5z = -2$

b) Find a vector W that is parallel to $V = i + 2j + 2k$ but of length 7.25.

$|V| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$W = \frac{7.25}{3} V = \frac{7.25}{3} i + \frac{14.5}{3} j + \frac{14.5}{3} k$

c) Find dz/dx when $x = 2$ and $y = 0$ where

$$e^{zxy} + 4zx - z^2 + xz^2 - 6z + y^2 - x^3 = 0$$

$$\frac{dz}{dx} = \frac{-f_x}{f_z} = -\frac{(zye^{zxy} + 4z + z^2)}{(xye^{zxy} + 4x - 2z + 2xz - 6)}$$

first find point

$e^0 + 8z - z^2 + 2z^2 - 6z = 0$

$1 + z^2 + 2z = 0$

$z^2 + 2z + 1 = 0$

$(z+1)(z+1) = 0$

$z = -1$

$$\left.\frac{dz}{dx}\right|_{(2,0,-1)} = -\frac{(-1(0)e^{zxy} - 4 + 1)}{(0 + 8 + 2 - 4 - 6)} = \frac{-3}{0}$$

undefined, vertical slope

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MTH 203, Calculus III, Test II Spring 2013

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QUESTION 1. Let $f(x, y) = x^2 + x + 2y + xy + y^2 + 3$

(i) Find $D_u(f)$ at the point (0, 0) in the direction of $v = i + 2j$

(ii) at the point (0, 0), in which direction does $D_u(f)$ obtain its maximum value? what is the maximum value?

(iii) Find an equation of the plane that contains the vector v as in (i) and the two points (0,0,0), (0,0,3).

(iv) Find an equation of the tangent plane to the solid object determined by $f(x, y)$ at the point (0, 0).

(v) The plane in (iii) intersects the solid object determined by $f(x, y)$ in a curve C. Find a parametric equations of the curve C (Write your answer in terms of x). Then Draw The Curve C.

(vi) Find a parametric equations of the tangent line, say L, to the curve C (as in v) at the point (0, 0) (write your solution in terms of x). Then find its slope.

(vii) The domain of the line in (vi) is a line, say F, in the xy plane. Find the acute angle between the two lines (i.e., between L and F).

QUESTION 2. Given : $5ze^{zx} + ln(y) - yz + zx - 20 = 0, x = 5t - 7s - 1, y = 4t - 6s + 1$.

1) Find dz/dt when $t = 3,\ s = 2$.

2) Find the equation of the tangent plane to the solid object above at the point (0, 1, 5). [Hint: You may use SOME of the calculations you already made in (i)]

QUESTION 3. Find all points in the xy-plane where the rate of change of z = 0 in the direction of the vector $i - j$, where $z = f(x, y) = 2x^2 + y^2 - xy + 4y + 1$

QUESTION 4. 1) Find an equation of the plane P that contains the line $< 2t - 1, 2t + 3, 5 + t >$ and the point $(1, 2, 5)$.

Does P contain the vector $< 3, -4, 1 >$? explain

QUESTION 5. Given $r(x) : x \in R, y = \sqrt{2}e^x, z = 0.5e^{2x} + 1$.

1) Find the arch length of $r(x)$ when $0 \leq x \leq ln(3)$.

2) What is the slope of the tangent line to the curve of $r(x)$ at the point $(0, \sqrt{2}, 1.5)$? [It is indeed nice to think!!!]

QUESTION 6. Let $f(x, y) = e^{x-2} - x + \frac{1}{3}y^3 - y^2 - 3y + 2$. Find the critical points of $f(x, y)$. Does $f(x, y)$ have local min. (max) values? if yes then find them.

QUESTION 7. Given the force field

$$F(x, y, z) = (yz^2 + 3y)i + (xz^2 + 3x - z)j + (2zxy - y + 10))k$$

.

1) Find Curl(F).

2) Is $F(x, y, z)$ conservative? If yes, find a function $L(x, y, z)$ such that $\nabla L(x, y, z) = F(x, y, z)$.

3) A particle moves along line segments from (0, 0, 1) to (1, 1 , 2) to (1, 2, 1). Find the work done by the above force $F(x, y, z)$ in moving the particle along the given line segments from (0, 0, 1) to (1, 2, 1)).

Faculty information

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MTH 203, Calculus III, Final EXAM spring 2013

Ayman Badawi

(THERE ARE 16 items, each item = 5 points, Total = 80 points)

QUESTION 1. a)Given $v =< 2, 2, 1 >$. Is there a vector u of length one such that $u \times v =< -2, 2, 0 >$? If no, then explain. If yes, then find such u.

b) Find the spherical coordinates and the cylindrical coordinates of the point $(-2, 2, -1)$

c) Given that the vector $u =< 3, 4 >$ lies in two distinct perpendicular planes P_1 and P_2 and the point $Q = (1, -4, 5)$ lies in both planes P_1 and P_2. Find an equation for P_1 and an equation for P_2. [Hint: there are infinitely many planes P_1, P_2 satisfy the given conditions, I only need two such planes]

QUESTION 2. Find a parametric equations of a line L that contains the point $p = (0, 0, 1)$ but it does not intersect the plane $P: \ z = 10 + 3x - 4y$. [Hint: there are infinitely many lines satisfy the given conditions, I only need one such line]

QUESTION 3. Given $xln(z) + zy + xy + 12x + 14y - 28 = 0$, $x = 3t - 2u$, $y = 2 - tu$. Find dz/dt when $t = u = 1$

QUESTION 4. Given that the point $Q = (2, 1, 4)$ lies on the surface (solid object) that is determined by a function $z = f(x, y)$. We know there are infinitely many tangent lines to the solid object $f(x, y)$ at the point Q. Given that $L : x \in R, y = 3x - b, z = 7x + c$ is a tangent line to the solid object $f(x, y)$ at the point Q. Find the values of b and c. Then find the directional derivative of the function $f(x, y)$ in the direction of the vector $i + 3j$ at the point Q.

QUESTION 5. Find the surface area of the part of $z = 2 + 2x^{3/2} + \sqrt{15}y$ over the region D that is located in the first quadrant of the xy plane and bounded by the line $y = 9x + 16$, the $x - axis$, where $0 \leq x \leq 1$.

QUESTION 6. Find all points at which the direction of the fastest change of the z-value for the function $z = f(x, y) = 0.5x^2 - 2x + y^2 - 6y$ is in the direction of $i + 2j$ [HINT: Recall If a vector U is in the direction of V, then $U = cV$ for some positive constant c]

QUESTION 7. Find an equation of the tangent plane to the surface $x^2 + xy + yz + z^2 + y - 3 = 0$ at the point $(-1, 1, 1)$.

QUESTION 8. If possible, find all Local min. and Local max. values of z where $z = f(x, y) = 7x - y^2e^x + 2ye^x - 8e^x$.

QUESTION 9. Given the force field $F(x, y) = yi + 6xj$. A particle moves along line segments from (0, 0) to (4, 0) to (6, 2) to (0, 2), then back to (0, 0) (counter clock-wise). Find the work done by the force $F(x, y)$ in moving the particle along the given line segments from (0, 0) back to (0, 0).

QUESTION 10. Given the force field $F(x, y, z) = yi + zj + 2yk$.

a) Find $Curl(F)$

b) A particle moves along the line segment from $(1, 1, 1) to (2, 3, 3)$. Find the work done by the above force $F(x, y, z)$ in moving the particle along the given line segment from (1, 1, 1) to (2, 3, 3).

c)A particle moves along line segments from (0, 0, 0) to the points $(1, 0, 0), (1, 2, 1), (0, 2, 1)$ and back to (0, 0, 0). Under the influence of the force above, find the work done.[Hint: Use Stoke's Theorem, first find an equation of a plane that contains the given points, write z in terms of x, y. Let S be the portion of the plane over the region D , where D is the rectangle in the xy-plane, $0 \leq x \leq 1$, $0 \leq y \leq 2$, note that dS = (-dz/dx)i + (- dz/dy)j + k and the work = $\int \int_D curl(F).dS$]

QUESTION 11. a) Find the volume of the solid object in 3D that has a basis D, where D is the region in the first quadrant of the xy-plane that is enclosed by the y-axis, the line y = 1 and the line $y = 0.5x - 4$. The height of the solid object is determined by $z = e^{(y^2+8y-8)}$.

b) Find the area of the side that is bounded by $z = f(x,y) = \sqrt[3]{\frac{9}{4}y^2} + 1$ and the curve $C : 0 \leq x \leq 1, y = \frac{2}{3}x^{3/2}$ in the xy-plane.

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